Behavior of the Economy that Necessitates Household Work

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Abstract

Not all goods and services consumed by household members are produced within the household, but they are supplemented by firms and the government. As the economy develops, it tends to be industrialized and urbanized. Furthermore, the population tends to be aging with nuclear families more numerous than multigenerational ones. All these developments combine to have made it more difficult for some households to adequately procure goods and services needed by their members, so that they are obliged to produce whatever amount of them in excess of external supplies. Thus, they are overworked.

A simple model of the economy that necessitates this type of "sociostructural overwork" is formulated to analyze its behavior over aggregate fluctuations. Then, it turns out that the degree of sociostructural overwork is countercyclical in the sense that it is alleviated during aggregate expansion while it is aggravated during aggregate contraction.

Keywords: child care, elderly care, household work, overwork, family welfare, aggregate fluctuations

INTRODUCTION

The fact that household sector is large is in controvertible. It employs 51 % and 40 % of the respective working age population in Japan and the United States (Statistics Bureau, Japan Management and Coordination Agency, 1976 to date; Bureau of Labor Statistics, U.S. Department of Labor, 1976 to date; Maruyama and Sasaki, 1998). Household expenditures for consumer services and nondurables constitute 47 % and 59 % of respective GDP in the two countries. Furthermore, household investment (e.g., Eisner, 1989; Greenwood and Hercowitz, 1991) in the form of consumer durables and semidurables, residential investment, and household equipment and structures (19 % and 12 % of respective GDP) exceeds firm investment in equipment and structures (14 % and 10 % of respective GDP in recent years (Economic Planning Agency of Japan, 1976 to date; Bureau of Economic Analysis,

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U.S. Department of Commerce, 1976 to date; Maruyama and Sasaki, 1998)). Its output of goods and services is estimated by Eisner (1989) to amount to 80 % of firm output, which is recently reconfirmed by Bryant, Zick and Kim (1992) in the United States.

In a similar study the Yoka Kaihatsu Center (Leisure Development Center, 1978) evaluated the value of household work at 40 % of the 1976 National Income of Japan, which was comparable to the Net National Product of manufacturing and service Industries in the same year. The Economic Planning Agency (1997) has recently estimated the value of unpaid household work at 15~23 % of 1996 GDP. However, their estimates are criticized for underestimation because they fail to include the time for learning, and the time for commuting to and form schools and factories (offices). The latter of which by many authors including Eisner (1989) and Greenwood and Hercowitz (1991) is thought of as gift from households to firms, though the Economic Planning Agency includes the time for volunteer activities.

Not all household goods and services are produced within the household, but they are supplemented by external supplies from firms and the public sectors including such informal ones as neighboring communities. As the economy develops, it becomes industrialized and urbanized. Furthermore, the population tends to be aging with the nuclear families more numerous than the multigenerational ones. All these developments combine to have made it more difficult for some households to adequately procure household goods and services needed by their members. Hence, these households are obliged to produce whatever amount of them needed by their members in excess of external supplies. Thus, they are obliged to work so hard as to render the value of marginal product of their household labor to fall short of the market wage rate. In other words they are simply overworked. This type of overwork by nature cannot be alleviated but rather aggravated to the contrary by reducing the aggregate demand as will be seen in the subsequent sections. Hence, it may be conveniently referred to as "sociostructural overwork" (Maruyama, 1994 and 1996).

This paper intends to analyze the behavior of such an economy as necessitates the sociostructural overwork. Actually, this task has been attempted by Maruyama (1996). However, it turns out that his work can be improved in several dimensions by the recent development of the internal price (wage) effects due to Sonoda and Maruyama (1999 and 2000). This fact has prompted the present project. The following section introduces the model, which naturally proves to be a revised version of the model due to Maruyama (1996), but is improved in several ways by incorporating the recent results relative to the "internal price (wage) effects." Section 3 analyzes the behavior of this economy over aggregate fluctuations, which are artificially generated by changes in autonomous investment. Section 4 discusses the results of this analysis. Finally, Section 5 concludes the paper.

A MODEL OF THE ECONOMY THAT NECESSITIES THE "SOCIOSTRUCTURAL OVERWORK"

The economy consists of m identical households, n identical firms, and a single government. The model is depicted graphically in Fig. 1. The outputs of these sectors are numbered respectively in the order of the household, the firm and the government sector. The output of a firm and the government

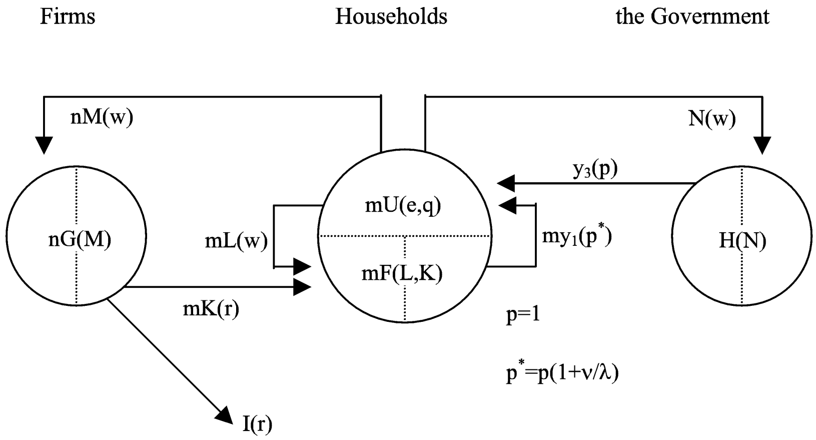

Fig. 1

are well-behaved functions of the employment of labor, M and N, respectively, while that of a household is a similar function of the employment of labor L and other inputs K.

$$y_1 = F(L,K) \quad y_2 = G(M) \quad \text{and} \quad y_3 = H(N)$$

Goods and services consumed by members of the households mq are produced by households and the government. It is assumed that whatever amount of them in excess of those supplied by the government is to be produced by households themselves.

$$mF(L,K) \geq mq - H(N) \quad \text{or} \quad mF(L,K) + H(N) - mq \geq 0 \tag{1}$$

A part of the output of firms nG(M) is supplied to households for other inputs mK into their production and the rest is used for investment I, which is assumed to be autonomous for simplicity.

$$nG(M) \geq mK + I \quad \text{or} \quad nG(M) - mK - I \geq 0\,, \quad I = \text{const} > 0 \tag{2}$$

Labor is supplied by households and is employed by themselves, firms and the government,

$$mLs - nM - N \geq 0\,, \quad L_s \equiv T - L - e\,, \quad T = \text{const} > 0 \tag{3}$$

where T and e, respectively, denote the endowment of time and the consumption of leisure by each household.

Each firm and the government are price takers and organize their production so as to maximize their profit $\pi_2 \equiv rG(M) - wM$ and $\pi_3 = pH(N) - wN$, respectively, where r and p denote the price of their outputs while w denotes the rate of wage. The government is assumed to make neither profit nor loss. Hence,

$$rG_1(M) - w \leq 0 \tag{4}$$

$$pH_1(N) - w \leq 0 \tag{5}$$

Each household is a price taker to satisfy the budget constraint (6) below as well as the constraint (1) requiring whatever amount of goods and services consumed by household members in excess of their government supply be produced by the household itself.

$$\pi_1 + wT - we - pq \geq 0, \quad \pi_1 \equiv pF(L, K) - wL - rK \tag{6}$$

It organizes its production so as to maximize its welfare, which is a well-behaved function of the consumption of leisure e, and goods and services q.

$$W = U(e, q)$$

The Kuhn-Tucker-Lagrange conditions associated with its welfare maximization require that the following relations be satisfied along with the inequalities (1) and (6) above.

$$p^* F_1(L, K) - w \leq 0, \quad p^* = p(1 + \nu/\lambda) \tag{7}$$

$$p^* F_2(L, K) - r \leq 0 \tag{8}$$

$$U_1(e, q) - \lambda w \leq 0 \tag{9}$$

$$U_2(e, q) - \lambda p^* \leq 0 \tag{10}$$

where λ denotes the Lagrange multiplier associated with the budget constraint (6) above, indicating the level of marginal household welfare of its full income (Becker, 1965), while ν denotes the Lagrange multiplier associated with the constraint (1) above, indicating the magnitude of the premium in terms of household welfare its members are prepared to pay for an additional unit of goods and services which are in short supply. The newly introduced p^* may be referred to as the "internal price" of goods and services which is greater than its market price p, since they are in short supply. The budget constraint (6) above is rewritten in terms of p^* in the following way.

$$p^* F(L, K) - wL - rK - we - p^* q + wT - pH(N) \geq 0 \tag{6a}$$

The system of inequalities (1)~(10) complete the model of our economy, where goods and services in the (external) market are conveniently chosen to be a numeraire. Then, p=1, though household members are willing to pay for a unit of these goods and services within the household a premium equal to ν/λ on top of it since they are in short supply.

Relations (4), (5), (7), and (8) combine to imply for interior solutions in which their equalities hold that

$$pF_1(\cdot) < p^* F_1(\cdot) = w = pH_1(\cdot) = rG_1(\cdot) \tag{11.1}$$

$$pF_2 < p^* F_2(\cdot) = r, \ p{=}1 \tag{11.2}$$

Since the external supply of household goods and services are not sufficient, the household is obliged to fill their deficiencies, which effort drives the market value of the marginal product of family labor below the market wage rate w, while its counterparts are equal to the market wage rate in other sectors. Thus, the differential in wage rate arises between the household and other sectors. Furthermore, relations (9) and (10) combine to imply for interior solutions that

$$pU_1(\cdot)/U_2(\cdot) < p^*U_1(\cdot)/U_2 = w\,,\ p=1 \tag{12}$$

Thus, it turns out that such heavy input of family labor is made possible by the extremely low supply price of labor $pU_1(\cdot)/U_2(\cdot)$, which is lower than the market wage rate. In other words, the household members want these goods and services so keenly that they are prepared to sacrifice any leisure sufficient to secure them. If it is possible to reduce employment of labor in the household sector and divert some to the government sector, the increase in its supply of these goods and services will more than offset the corresponding decrease in their supply by the household sector since the marginal product of government labor is greater than that of household labor. Hence, the level of household welfare will be raised.

BEHAVIOR OF THE ECONOMY THAT NECESSITIES THE "SOCIOSTRUCTURAL OVERWORK"

Now the behavior of this economy over aggregate fluctuations will be examined to better understand implications of the "sociostructural overwork" within the household sector. Since investment is autonomous, we can change it artificially to generate aggregate fluctuations via its multiplier effect. Responses of each sector to aggregate fluctuations will be examined by means of the comparative statics analysis of the system of inequalities (1)~(10) for interior solutions in which their equalities hold. The associated comparative statics analysis is described compactly in the matrix expression (13).

$$\begin{bmatrix} 0 & 0 & nG_1 & 0 & 0 & -m & 0 & 0 & 0 & 0 \\ 0 & 0 & -n & -1 & -m & 0 & -m & 0 & 0 & 0 \\ G_1 & -1 & rG_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & H_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & p^*F_{11} & p^*F_{12} & 0 & 0 & 0 & F_1 \\ -1 & 0 & 0 & 0 & p^*F_{21} & p^*F_{22} & 0 & 0 & 0 & F_2 \\ 0 & -\lambda & 0 & 0 & 0 & 0 & U_{11} & U_{12} & -w & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & U_{21} & U_{22} & -p^* & -\lambda \\ -mK & mL_s & 0 & \Delta pH_1 & 0 & 0 & -mw & -mp^* & 0 & 0 \\ 0 & 0 & 0 & H_1 & mF_1 & mF_2 & 0 & -m & 0 & 0 \end{bmatrix} \begin{bmatrix} dr \\ dw \\ dM \\ dN \\ dL \\ dK \\ de \\ dq \\ d\lambda \\ dp^* \end{bmatrix} = \begin{bmatrix} dI \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{13}$$

$$L_s \equiv T - L - e\,;\ p=1,\ \ p^* \equiv p(1-\nu/\lambda)\,.\ \ \Delta p \equiv p^* - p$$

The Jacobian determinant A formed by the coefficients of structural variables on its left hand side proves to be negative as shown in Supplement 2 and makes it possible to solve these equalities uniquely for the structural variables. To give an overview of their solutions, results for major variables will be briefly reviewed.

1) Employment and Output

a) The firm and the government sectors

$$\frac{dM}{dI}A = -mH_{11}\frac{\partial L_S}{\partial r}C - m\eta_w(M)MG_{11}\eta_w(N)NH_{11}\frac{\partial L_S}{\partial H}C$$

$$-m\eta_w(M)MG_{11}\frac{\partial L_S}{\partial w}C + \eta_w(M)MG_{11}C(<)0,$$

$$\frac{dM}{dI}(>)0 \qquad (14.1)$$

$$A(<)0, \quad C>0; \ \eta_X(Z) \equiv \frac{\partial Z}{\partial X}\frac{X}{Z}, \text{X=w; Z=M, N}$$

$$\therefore \frac{dy_2}{dI} = \frac{d}{dI}G(M) = G_1\frac{dM}{dI}(>)0 \qquad (14.2)$$

$$\frac{dN}{dI}A = -n\eta_w(M)MG_{11}C - mrG_{11}\frac{\partial L_S}{\partial r}C(<)0, \quad \frac{dN}{dI}(>)0 \qquad (14.3)$$

$$\therefore \frac{dy_3}{dI} = \frac{d}{dI}H(N) = H_1\frac{dN}{dI}(>)0 \qquad (14.4)$$

Here, C denotes the bordered Hessian determinant associated with the comparative statics analysis of the optimal choice of each household, which is described by the system of inequalities (**1**), (**6**)~(**10**), and proves to be positive as shown in Supplement **1**.

b) The household sector

$$\frac{dL}{dI}A = -n\eta_w(M)MG_{11}\eta_w(N)NH_{11}\frac{\partial L}{\partial H}C - \eta_w(M)MG_{11}H_{11}\frac{\partial L}{\partial w}C$$

$$-(rG_{11}+nH_{11})\frac{\partial L}{\partial r}C - mr\eta_w(N)NH_{11}G_{11}\frac{\partial^2(L,e)}{\partial r\partial H}C$$

$$-mrG_{11}H_{11}\frac{\partial^2(L,e)}{\partial r\partial w}C(>)0, \quad \frac{dL}{dI}(<)0 \qquad (15.1)$$

$$\frac{dK}{dI}A = -n\eta_w(M)MG_{11}\eta_w(N)NH_{11}\frac{\partial K}{\partial H}C - n\eta_w(M)MG_{11}H_{11}\frac{\partial K}{\partial w}C$$

$$-(rG_{11} - nH_{11})\frac{\partial K}{\partial r}C - mrG_{11}\eta_w(N)NH_{11}\frac{\partial^2(L_S, K)}{\partial r \partial H}C$$

$$-mrG_{11}H_{11}\frac{\partial^2(L_S, K)}{\partial r \partial w}C(>)0, \quad \frac{dK}{dI}(<)0 \tag{15.2}$$

$$\therefore \frac{dy_1}{dI} = \frac{d}{dI}F(L, K) = F_1\frac{dL}{dI} + F_2\frac{dK}{dI}(<)0 \tag{15.3}$$

Here, the expression $\partial^2(L,e)C/\partial r\partial H$ is formed out of the bordered Hessian determinant C, by replacing the two columns corresponding to dL and de respectively by the two columns representing the coefficients of dr and dH respectively on the right hand side of (S**1**) below in Supplement **1**. It represents the effect of combined changes in the price r of firm output and in the supply H of household goods and services by the government on the combination of the household employment and leisure. $\partial^2(L,e)/\partial r\partial w$ is similarly formed. Furthermore, $\partial^2(L_S,K)C/\partial r\partial H$ denotes the sum of $\partial^2(-L,K)C/\partial r\partial H$ and $\partial^2(-e,K)C/\partial r\partial H$. $\partial^2(L_S,K)C/\partial r\partial w$ is also similarly formed.

2) Prices, Wage Rates, and Internal Prices and Rates

$$\frac{dr}{dI}A = -(rG_{11} + nH_{11})C + mrG_{11}\eta_w(N)NH_{11}\frac{\partial L_S}{\partial H}C$$

$$+mrG_{11}\frac{\partial L_S}{\partial w}C(>)0, \frac{dr}{dI}(<)0 \tag{16.1}$$

$$\frac{dw}{dI}A = -n\eta_w(M)MG_{11}H_{11}C + mrG_{11}H_{11}\frac{\partial L_S}{\partial r}C(>)0, \quad \frac{dw}{dI}(<)0 \tag{16.2}$$

$$\frac{dp^*}{dI}A = -nG_1\frac{\partial p^*}{\partial H}C - nG_1H_1\frac{\partial p^*}{\partial w}C - (rG_{11} + nH_{11})\frac{\partial p^*}{\partial r}C$$

$$-mrG_{11}\frac{\partial^2(p^*, L_S)}{\partial r \partial H}C - mrG_{11}H_{11}\frac{\partial^2(p^*, L_S)}{\partial r \partial w}C(>)0,$$

$$\frac{dp^*}{dI}(<)0 \tag{16.3}$$

$$\frac{d}{dI}pF_1(L,K)=F_{11}\frac{dL}{dI}+F_{12}\frac{dK}{dI}(>)0,\quad p=1 \qquad (16.4)$$

From relation (4) for interior solutions in which its equality holds, $\ln r+\ln G_1(M)=\ln w$,, hence

$$\frac{1}{r}\frac{dr}{dI}+\frac{1}{G_1}G_{11}\frac{dM}{dI}=\frac{1}{w}\frac{dw}{dI},\quad \frac{1}{G_1}G_{11}\frac{dM}{dI}(<)0$$

$$\therefore \frac{1}{r}\frac{dr}{dI}(>)\frac{1}{w}\frac{dw}{dI};\quad \frac{1}{r}\frac{dr}{dI}(<)0,\quad \frac{1}{w}\frac{dw}{dI}(<)0$$

$$\therefore \left|\frac{1}{r}\frac{dr}{dI}\right|(<)\left|\frac{1}{w}\frac{dw}{dI}\right| \qquad (16.5)$$

Furthermore, from relation (7) for interior solutions in which its equality holds, $\ln p^*+\ln F_1(L,K)=\ln w$, hence

$$\frac{1}{p^*}\frac{dp^*}{dI}+\frac{1}{F_1}\left(F_{11}\frac{dL}{dI}+F_{12}\frac{dK}{dI}\right)=\frac{1}{w}\frac{dw}{dI}(<)0,\quad F_{11}\frac{dL}{dI}+F_{12}\frac{dK}{dI}(>)0$$

$$\therefore \frac{1}{p^*}\frac{dp^*}{dI}<\frac{1}{w}\frac{dw}{dI},\quad \left|\frac{1}{p^*}\frac{dp^*}{dI}\right|(>)\left|\frac{1}{w}\frac{dw}{dI}\right| \qquad (16.6)$$

Therefore,

$$0=\left|\frac{1}{p}\frac{dp}{dI}\right|(<)\left|\frac{1}{r}\frac{dr}{dI}\right|(<)\left|\frac{1}{w}\frac{dw}{dI}\right|(<)\left|\frac{1}{p^*}\frac{dp^*}{dI}\right|,\ p=1 \qquad (16.7)$$

Since the market value of the marginal product of labor represents a reward to the labor employed, that of household labor $pF_1(\cdot)$ may be used for its wage rate. Then, the difference $w\text{-}pF_1(\cdot)$, $p=1$ measures the wage differential between the household and other sectors.

$$\frac{d}{dI}(w-F_1(L,K))=\frac{dw}{dI}-F_{11}\frac{dL}{dI}-F_{12}\frac{dK}{dI}(<)0,\quad F_{11}\frac{dL}{dI}+F_{12}\frac{dK}{dI}(>)0 \qquad (16.8)$$

3) Consumption, Labor Supply and Household Welfare

$$\frac{dq}{dI}A=-n\eta_w(M)MG_{11}\eta_w(N)NH_{11}\frac{\partial q}{\partial H}C-n\eta_w(M)MG_{11}H_{11}\frac{\partial q}{\partial w}C$$

$$-(rG_{11}-nH_{11})\frac{\partial q}{\partial r}C-mrG_{11}\eta_w(N)NH_{11}\frac{\partial^2(L_S,q)}{\partial r\partial H}C,$$

$$-\,mrG_{11}H_{11}\frac{\partial^2(L_S,q)}{\partial r\partial w}C(<)0,\ \ \frac{dq}{dI}(>)0 \tag{17.1}$$

$$\frac{de}{dI}A=-n\eta_w(M)MG_{11}\eta_w(N)NH_{11}\frac{\partial e}{\partial H}C-n\eta_w(M)MG_{11}H_{11}\frac{\partial e}{\partial w}C$$

$$-(rG_{11}+nH_{11})\frac{\partial e}{\partial r}C+mrG_{11}\eta_w(N)NH_{11}\frac{\partial^2(L,e)}{\partial r\partial H}C,$$

$$+mrG_{11}H_{11}\frac{\partial^2(L,e)}{\partial r\partial w}C(<)0,\ \ \frac{de}{dI}(>)0 \tag{17.2}$$

$$\therefore m\frac{dL_S}{dI}=m\frac{d}{dI}(T-e)=m\frac{dL}{dI}+n\frac{dM}{dI}+\frac{dS}{dI}(<)0 \tag{17.3}$$

$$\frac{d\lambda}{dI}A=-n\eta_w(M)MG_{11}\eta_w(N)NH_{11}\frac{\partial\lambda}{\partial H}C-n\eta_w(M)MG_{11}H_{11}\frac{\partial\lambda}{\partial w}C$$

$$-(rG_{11}+nH_{11})\frac{\partial\lambda}{\partial r}C+mrG_{11}\eta_w(N)NH_{11}\frac{\partial^2(L_S,\lambda)}{\partial r\partial H}C$$

$$+mrG_{11}H_{11}-rG_{11}\frac{\partial\lambda}{\partial r}C+mrG_{11}\eta_w(N)NH_{11}\frac{\partial^2(L_S,\lambda)}{\partial r\partial H}C$$

$$+mrG_{11}H_{11}\frac{\partial^2(L_S,\lambda)}{\partial r\partial w}C(>)0,\ \ \frac{d\lambda}{dI}(<)0 \tag{17.4}$$

Since leisure, and goods and services are normal, full consumption Z of the household is equal to its full income y (Becker, 1965).

$$Z\equiv we+p^*q=\pi_1+wT=y$$

Furthermore, relations (9) and (10) above imply for interior solutions in which their equalities hold that

$$\therefore\frac{dW}{dZ}=\frac{dW}{dy}=\lambda>0$$

From relation (17.4)

$$\frac{d}{dI}\left(\frac{dW}{dZ}\right)=\frac{d}{dI}\left(\frac{dW}{dy}\right)=\frac{d\lambda}{dI}(<)0$$

Thus in case the household welfare W is concave,

$$\frac{d^2W}{dZ^2}=\frac{d^2W}{dy^2}<0 \quad \text{and} \quad \frac{dZ}{dI}=\frac{dy}{dI}(>)0 \tag{17.5}$$

$$\frac{dW}{dI}=\frac{dW}{dZ}\frac{dZ}{dI}=\frac{dW}{dy}\frac{dy}{dI}(>)0 \tag{17.6}$$

EXAMINATION OF THE BEHAVIOR OF THE ECONOMY THAT NECESSITATES THE "SOCIOSTRUCTURAL OVERWORK"

In relations (1) and (6)~(10) above specifying the optimal household choice, relations (7) and (8) directly associated with the organization of household production share the internal price p^* of household goods and services with relations (9) and (10) directly associated with the choice of household consumption. Thus, these relations are indecomposable (e.g., Maruyama, 1984; Singh, Squire and Strauss, 1986) in the sense that the household production and consumption are to be jointly determined. The indecomposability of its optimal choice has several significant effects on its comparative statics analysis. Both the "internal price effects" (Sonoda and Maruyama, 1999 and 2000) and the income effects inherent to its consumption choice creep into the determination of production organization so as to make the sign of the responses of many structural variables indeterminate. However, in case all economic units are competitive with no residual profits imputed and their technology (welfare functions) are of the Cobb-Douglas type, many of these indeterminacies are resolved as shown in Supplements 3 and 4. Several inequalities are parenthesized in the preceding section to show that such cases are addressed. It may be more enlightening to limit our examination to such cases hereafter.

Employment and output of the firm and the government sectors are procyclical in the sense that they fluctuate in accordance with the fluctuations of aggregate demand that are artificially generated by the changes in investment as shown in relations (14.1)~(14.4). It is noteworthy that output of the government sector expands, the demand for which is not increased directly by the rise in investment. This can be seen as follows. As investment rises, output therefore employment of firms increases, which in turn raises the wage income of households due to relation (11) $pF_1(\cdot)<w$. Increased income then enable them to satisfy their intended demand for household goods and services supplied by the government sector. This is what actually has resulted. On the other hand, both employment of labor and other inputs of households decrease as aggregate demand expands. Since both labor and other inputs are underemployed as shown in relations (11.1) and (11.2) above, households have easily lost in the demand for them to investment and other sectors. Hence their output decreases as well, which in turn expands their demand for goods and services supplied by the government sector. Whereas, all

these variables increase as aggregate demand contracts due to fall in investment as shown in relations (15.1)~(15.3). Thus, they are countercyclical.

As aggregate demand expands, both the price of firm output and the wage rate fall relative to the market price of household goods and services that are chosen as numeraire. The magnitude of fall in the latter proves to be greater than that in the former as shown in relations (16.1)~(16.5). However, since the choice of numeraire is arbitrary, this result may be thought of as indicating that the prices of other goods and services rise relative to that of labor, whose supply price is lower than the market wage rate as shown in relation (12). On the other hand, the market value of the marginal product of household labor rises, though it is lower than the market wage rate as shown in relation (16.4). This is mainly because the employment of household labor is reduced, which so far has been underemployed as shown in relation (11.1) above. Furthermore, the internal price of household goods and services falls, though it is higher than its market counterpart, since the increase in their external supply from the government sector outweighs the decrease in their supply within the household so as to enable their members to consume more of them as shown in relation (17.1) below. Thus, the price (wage) differentials of the two within and without the household contract as aggregate demand expands, which in effect indicates that a better allocation of resources has been achieved as shown in relations (16.7) and (16.8).

Both leisure and consumption of goods and services increase as aggregate demand expands as shown in relations (17.1) and (17.2). They have been made possible by increased wage income due to expanded employment in other sectors. Households take advantage of their increased purchasing power in consumption of leisure as well as of goods and services. Thus, both the overwork of household members and the shortage of goods and services within the household are alleviated, which are summarized in the increased level of household welfare in case it is a concave function of leisure, and goods and services an shown in relations (17.3)~(17.6). The higher Pareto efficiency is achieved due to a better allocation of labor among relevant sectors. It is a matter of course that responses of the structural variables are made in the other direction in case aggregate demand contracts. Finally, it is to be noted that the “sociostructural overwork” has some aspects closely connected with aggregate fluctuations, since it is alleviated during aggregate expansion while it is aggravated during aggregate contraction.

CONCLUSION

The household sector is incontrovertibly large. It employs 51 % and 40 % of the respective working age population in Japan and the United State. Household expenditures for consumer services and nondurables constitute 47 % and 59 % of respective GDP. Furthermore, household investment (e.g., Eisner, 1989; Greenwood and Hercowitz, 1991) in the form of consumer durables and semidurables, residential investment, household equipment and structures exceeds firm investment in equipment and structures in the two countries in recent years. Its output of goods and services is estimated to amount to 80 % of firm output in the United States. Similar estimates have been made by the Yoka Kaihatsu Center (1978) in Japan.

Not all household goods and services are produced within the household, but are supplemented by external supplies from firms and the public sectors including such informal ones as neighboring communities. As the economy develops, it becomes industrialized and urbanized. Furthermore, the population tends to be aging with the nuclear families more numerous than the multigenerational ones. All these developments combine to have made it more difficult for some households to adequately procure household goods and services needed by their members. Hence, these households are obliged to produce within the household whatever amount of them needed by their members in excess of external supplies. Thus they are obliged to work so hard as to lower the value of marginal product of their labor below the market wage rate. They are simply overworked. This type of overwork cannot naturally be alleviated by reducing aggregate demand therefore aggregate employment, hence it may be conveniently referred to as the "sociostructural overwork" (Maruyama, 1994 and 1996).

A simple model of such an economy as necessitates the "sociostructural overwork" is formulated and analyzed to see its behavior over aggregate fluctuations which are artificially generated by changes in autonomous investment. Responses of the household sector involve many indeterminacies, in which their production organization and consumption choice are indecomposable (Maruyama, 1984; Singh, Squire and Strauss, 1986). Nonetheless, the specification of the production and welfare functions of the Cobb-Douglas type resolves most of these indeterminacies. Employment and output of the firm and public sectors are procyclical in the sense they fluctuate in accordance with aggregate fluctuations. As aggregate demand expands, both employment and output of these sectors expand. Hence, both goods and services directly consumed by household members and those put into their production are increased. Furthermore, employment by these sectors expands, which increases the wage income earned by the households to enable them to procure more of goods and services produced by these sectors. On the other hand, both employment of labor and other inputs by households contract. So naturally does their output. Thus, their overwork is alleviated and their demand for goods and services produced by other sectors is increased, which in turn is to be satisfied thanks to their increased wage income.

The differentials both in the price of household goods and services and in the wage rate (the market value of marginal product of labor) within and without the households are narrowed, which implies a better allocation of labor, and goods and services. Consumption of household goods and services expands, which is made possible by the increases both in their external supply and in the wage income earned by the households. Their consumption of leisure expands as well since the decrease in the household employment outweighs the corresponding increase in the wage employment by other sectors. Hence, the household welfare is enhanced, which is an increasing function of the consumption of leisure, and goods and services. The higher Pareto efficiency realized in this phase of aggregate fluctuations is due to a better allocation of labor, and goods and services which is reflected in the narrowed differentials in the price and wage rate within and without the households. After all these examinations, it turns out that the sociostructural overwork within the household is not alleviated by the contraction but by the expansion of aggregate demand therefore aggregate employment. In this sense the degree of "sociostructural overwork" is countercyclical.

Supplement 1

$$\begin{bmatrix} p^*F_{11} & p^*F_{12} & 0 & 0 & 0 & F_1 \\ p^*F_{21} & p^*F_{22} & 0 & 0 & 0 & F_2 \\ 0 & 0 & U_{11} & U_{12} & -w & 0 \\ 0 & 0 & U_{21} & U_{22} & -p^* & -\lambda \\ 0 & 0 & -mw & -mp^* & 0 & 0 \\ mF_1 & mF_2 & 0 & -m & 0 & 0 \end{bmatrix} \begin{bmatrix} dL \\ dK \\ de \\ dq \\ d\lambda \\ dp^* \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & -\lambda \\ 0 & 0 & 0 \\ \Delta p & -mK & mL_S \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} dH \\ dr \\ dw \end{bmatrix} \qquad \text{(S1)}$$

$$p^* \equiv p(1-\nu/\lambda), \quad \Delta p \equiv p^* - p; \quad L_S \equiv T - L - e$$

Supplement 2

$$A = nG_1A_{13} - mA_{16} = nG_1\frac{dM}{dI}A - m\frac{dK}{dI}A(<)0 \qquad \text{(S2)}$$

Supplement 3

$$\frac{dN}{dI} = -n\eta_w(M)MG_{11}C - mrG_{11}\frac{\partial L_S}{\partial r}C(<)0; \quad A(<)0, C>0$$

The right hand side of this equation is divided into three groups of terms to facilitate its evaluation.

$$(1)\ -n\eta_w(M)MG_{11}C = -nG_1m^2p^{*2}\{F_2(F_2F_{11}-F_1F_{21}) - F_1(F_2F_{12}-F_1F_{22})\}(p^*U_{11}-wU_{21})$$
$$+nG_1m^2p^*w\{F_2(F_2F_{11}-F_1F_{21}) - F_1(F_2F_{12}-F_1F_{22})\}(p^*U_{12}-wU_{22})$$
$$-nG_1m^2p^{*2}w^2|F|\lambda < 0, \quad |F| \equiv F_{11}F_{22} - F_{12}F_{21} > 0$$

$$(2)\ mrG_{11}\frac{\partial L}{\partial r}C = m^3rG_{11}F_1F_2\{p^*(p^*U_{11}-wU_{21}) - w(p^*U_{12}-wU_{22})\}(>0)$$
$$-m^3rG_{11}Kp^*(F_2F_{12}-F_1F_{22})(p^*U_{11}-wU_{21})(<0) >< 0$$

$$(3)\ mrG_{11}\frac{\partial e}{\partial r}C = -m^3rG_{11}Kp^*\{F_2(F_2F_{11}-F_1F_{21}) - F_1(F_2F_{12}-F_1F_{22})\}(p^*U_{12}-wU_{22})(<0)$$
$$+m^3rG_{11}Kp^{*2}w|F|\lambda(<0) + m^3rG_{11}p^{*2}wF_2F_{11}\lambda(>0) >< 0$$

The first group of terms proves to be negative owing both to the normality of leisure, and household goods and services and to the concavity of household production functions. The first two positive terms of the second group are more than offset by the first two negative terms of the first group in case both households and firms are competitive and their technologies are of the Cobb-Douglas type: $F(L,K)=aL^{\alpha}K^{\beta}$, $\alpha+\beta<1$; $G(M)=a^*M^{\alpha^*}$, $\alpha^*<1$. For instance, evaluate the sum S of the first terms of these two groups.

$$S = m^2p^*r\{-nG_1(F_2F_{11} - F_1F_{21}) + mG_{11}F_1F_2\}(p^*U_{11} - wU_{21})$$

Put the expression in the brace as S^*, then

$$S^* = \left[\frac{-nG_1}{MG_{11}}\frac{F_2F_{11} - F_1F_{21}}{K|F|} + \frac{mG_{11}}{MG_{11}}\frac{F_1F_2}{K|F|}\right]MG_{11}K|F|$$

For the Cobb-Douglas production functions, S^* is rewritten as follows,

$$S^* = \left[\frac{n}{1-\alpha^*}\frac{-1}{1-\alpha-\beta} + \frac{m}{M}\frac{L}{1-\alpha-\beta}\right]MG_{11}K|F| > 0$$

which is commonly evaluated to be positive in the present day economy, where mL is nearly equal to nM. Then, S proves to be negative since $m^2p^*r(p^*U_{11}\text{-}wU_{21})<0$. Similarly, so does the sum of the second terms of these groups.

There is another positive term at the end of the third group. Make the sum S of this term and the final term of the first group.

$$S = m^2p^{*2}w\lambda\left\{\frac{-nG_1}{MG_{11}}\frac{w|F|}{K|F|} + \frac{mrG_{11}}{MG_{11}}\frac{F_2F_{11}}{K|F|}\right\}MG_{11}K|F|$$

The expression S^* in the brace is rewritten for the Cobb-Douglas production functions as follows,

$$S^* = \frac{nw}{(1-\alpha^*)K} - \frac{mr(1-\alpha)}{M(1-\alpha-\beta)} = \frac{nwM(1-\alpha-\beta) - mrK(1-\alpha)(1-\alpha^*)}{KM(1-\alpha^*)(1-\alpha-\beta)} > 0,$$

which is commonly evaluated to be positive from inequality (2) above in the second section and the competitiveness of firms so that nwM=mrK+rI. Then, S proves to be negative since $m^2p^{*2}w\lambda MG_{11}K|F|<0$. Thus, so does the sum of all terms of the three groups since all other terms remain to be negative. Hence, dN/dI proves to be positive since A(<)0.

Supplement 4

$$\frac{dL}{dI}A = -n\eta w(M)MG_{11}\eta_W(N)NH_{11}\frac{\partial L}{\partial H}C - \eta_W(M)MG_{11}H_{11}\frac{\partial L}{\partial w}C$$

$$+ (rG_{11} + nH_{11})\frac{\partial L}{\partial r}C - mr\eta_W(N)NH_{11}G_{11}\frac{\partial^2(L,e)}{\partial r\partial H}C$$

$$+ mrG_{11}H_{11}\frac{\partial^2(L,e)}{\partial r\partial w}C(>)0;\quad A(<)0,\quad C > 0$$

The right hand side of this equation is divided into six groups of terms to facilitate its evaluation.

$$(1)\quad -n\eta_W(M)MG_{11}\eta_W(N)NH_{11}\frac{\partial L}{\partial H}C$$

$$=-mnG_1H_1p^*(F_2F_{12}-F_1F_{22})\{(p^*U_{11}-wU_{21})-w(p^*U_{12}-wU_{22})\}>0$$

$$(2)\quad -\eta_W(M)MG_{11}H_{11}\frac{\partial L}{\partial w}C$$

$$=-nG_1H_{11}m^2F_2^2\{p^*(p^*U_{11}-wU_{21})-w(p^*U_{12}-wU_{22})\}(<0)$$

$$-nG_1H_{11}m^2p^{*2}wF_2F_{12}\lambda+nG_1H_{11}m^2L_Sp^*(F_2F_{12}-F_1F_{22})(p^*U_{11}-wU_{21})(>0)><0$$

$$(3)\quad rG_{11}\frac{\partial L}{\partial r}C=rG_{11}m^2F_1F_2\{p^*(p^*U_{11}-wU_{21})-w(p^*U_{12}-wU_{22})\}(>0)$$

$$+rG_{11}m^2p^*wF_{12}\lambda-rG_{11}m^2Kp^*(F_2F_{12}-F_1F_{12})(p^*U_{11}-wU_{21})(<0)><0$$

$$(4)\quad nH_{11}\frac{\partial L}{\partial r}C=nH_{11}m^2F_1F_2\{p^*(p^*U_{11}-wU_{21})-w(p^*U_{12}-wU_{22})\}(>0)$$

$$+nH_{11}m^2p^*w^2F_{12}\lambda-nH_{11}m^2Kp^*(F_2F_{12}-F_1F_{22})(p^*U_{11}-wU_{21})(<0)><0$$

$$(5)\quad -mr\eta_W(N)NH_{11}G_{11}\frac{\partial^2(L,e)}{\partial r\partial H}C=-m^2rG_{11}H_1\{\Delta pF_1F_2(p^*U_{12}-wU_{22})-p^*wF_{12}\lambda\}(>0)$$

$$+m^2rG_{11}H_1Kp^*(F_2F_{12}-F_1F_{22})(p^*U_{12}-wU_{22})(<0)><0$$

$$(6)\quad mrG_{11}H_{11}\frac{\partial^2(L,e)}{\partial r\partial w}C=$$

$$-m^3rG_{11}H_{11}Kp^{*2}F_2F_{12}\lambda(<0)-m^3rG_{11}H_{11}KF_2^2(p^*U_{12}-wU_{22})(>0)$$

$$+m^3rG_{11}H_{11}L_Sp^*wF_{12}\lambda(>0)-m^3rG_{11}H_{11}L_SF_1F_2(p^*U_{12}-wU_{22})(<0)><0$$

There are twelve positive terms and nine negative terms in all. Two negative terms of the second group turn out to more than offset respectively by the corresponding two positive terms of the third group. Make the sum S of the respective first terms of these groups.

$$S=m^2p^*F_2\{-nG_1H_{11}F_2+rG_{11}F_1\}(p^*U_{11}-wU_{21})$$

Then, the expression S^* in the brace will be rewritten as follows.

$$S^*=\left\{\frac{-nG_1}{MG_{11}}\frac{H_{11}F_2}{H_{11}N}+\frac{rG_{11}}{MG_{11}}\frac{F_1}{H_{11}N}\right\}MG_{11}\cdot NH_{11}$$

$$=\left\{\frac{-nG_1}{MG_{11}}\frac{1}{N}+\frac{1}{M}\frac{H_1}{NH_{11}}\right\}\frac{rMG_{11}NH_{11}}{p^*}$$

due to relations (5), (7) and (8) above in the second section for interior solutions in which they hold in equality. For the production functions of the Cobb-Douglas type: $G(M)=a^*M^{\alpha^*}$, $\alpha^*<1$ and $H(N)=a^{**}N^{\alpha^{**}}$, $\alpha^{**}<1$, S^* is evaluated as follows.

$$S^* = \left\{-\frac{nM}{(\alpha^*-1)NM} + \frac{N}{(\alpha^{**}-1)MN}\right\}\frac{rMG_{11}NH_{11}}{p^*},$$

which commonly proves to be negative in the present day economy, where $nM>N$ and $\alpha^* \fallingdotseq \alpha^{**}$. Then S proves to be positive, since $(p^*U_{11}-wU_{21})<0$. Similarly, so does the sum of the respective second terms of these groups.

Then, the third term of the third group which is negative is just offset by the second term of the fifth group, so is the third term of the fourth group by the third term of the second group due to relations (4), (5) and (8) above in the second section for interior solutions. The final term of the third group which is negative is more than offset by the corresponding term of the second group. Make the sum S of these two terms.

$$S = m^2\{nG_1H_{11}L_S - rG_{11}K\}p^*(F_2F_{12} - F_1F_{22})(p^*U_{11} - wU_{21})$$

The expression S^* in the brace is rewritten as follows.

$$S^* = \left\{\frac{nG_1}{MG_{11}}\frac{H_{11}mwL_S}{H_{11}N} - \frac{mrKG_{11}}{MG_{11}}\frac{H_1}{H_{11}N}\right\}\frac{MG_{11}NH_{11}}{mw},\ H_1=w$$

For the Cobb-Douglas production functions S^* is evaluated in the following way.

$$S^* = \frac{mwMmL_S}{(\alpha^*-1)NM} - \frac{mrK\cdot N}{(\alpha^{**}-1)NM} < 0,$$

which commonly proves to be negative in the present day economy in case firms are competitive, so that $nwM>mrK$, $mL_S=nM+N>N$ and $\alpha^* \fallingdotseq \alpha^{**}$ due to relations (2), (3) and (4) above in the second section for interior solutions. Hence, so does S since $m^2p^*(F_2F_{12}-F_1F_{22})(p^*U_{11}-wU_{21})<0$.

Now the final term of the fourth group which is negative is just offset by the first term of the same group, which can been seen as follows. Make the sum S of these two terms.

$$S = m^2nH_{11}\{F_1F_2 - K(F_2F_{12} - F_1F_{22})\}p^*(p^*U_{11} - wU_{21})$$

The expression S^* in the brace is rewritten as

$$S^* = \left\{\frac{F_1F_2}{L|F|} - \frac{K(F_2F_{12} - F_1F_{22})}{L|F|}\right\}L|F|,\ \ |F| \equiv F_{11}F_{22} - F_{12}F_{21} > 0,$$

which proves to vanish for the Cobb-Douglas technology, since

$$S^* = \left\{\frac{K}{1-\alpha-\beta} - \frac{K}{1-\alpha-\beta}\right\}L|F| = 0.$$

The final term of the fifth group which is negative is more than offset by the second term of the first group. Make the sum S of these two terms.

$$S = mH_1\{nG_1w + mrG_{11}K\}p^*(F_2F_{12} - F_1F_{22})(p^*U_{12} - wU_{22})$$

The expression S^* in the brace is rewritten as

$$S^* = \left\{\frac{nG_1w}{MG_{11}} + \frac{mrKG_{11}}{MG_{11}}\right\}MG_{11},$$

which is evaluated to be positive for the Cobb-Douglas technology

$$S^* = \left\{\frac{nwM}{(\alpha^* - 1)M} + \frac{mrK(\alpha^* - 1)}{M(\alpha^* - 1)}\right\}MG_{11} > 0,$$

since nwM>mrK in case firms are competitive. Hence S proves to be positive since $m^2nH_{11}p^*(p^*U_{11}\text{-}wU_{21})>0$.

The first term of the final group which is negative is more than offset by the third term of the same group,which can be seen as follows. Make the sum S of these two terms.

$$S = m^3rG_{11}H_{11}\{-Kp^*F_2F_{12} + L_SwF_{12}\}p^*\lambda$$

$$= m^3rG_{11}H_{11}\{-KF_2F_{12} + L_SF_1F_{12}\}p^{*2}\lambda \quad \therefore p^*F_1 = w$$

The expression S^* in the brace is rewritten as

$$S^* = \left\{-\frac{KF_1F_{12}}{KL|F|} + \frac{L_SF_1F_{12}}{LK|F|}\right\}KL|F|, \quad |F| \equiv F_{11}F_{22} - F_{12}F_{21},$$

which is evaluated for the Cobb-Douglas technology in the following way.

$$S^* = \left\{-\frac{K\beta}{K(1-\alpha-\beta)} + \frac{L_S \cdot \alpha}{L(1-\alpha-\beta)}\right\}KL|F|.$$

Thus, S^* commonly proves to be positive in the present day economy where $L_S \geq L$ and $\alpha>\beta$. Hence S in turn proves to be positive since $m^3rG_{11}H_{11}p^*\lambda>0$.

Finally the third term of the final group which is negative is more than offset by the second term of the fourth group. Make the sum S of these two terms.

$$S = -m^2H_{11}F_1F_2\{nw + mrG_{11}L_S\}(p^*U_{12} - wU_{22}).$$

The expression S^* in the brace is rewritten as

$$S^* = \left\{\frac{nrG_1}{MG_{11}} + \frac{mrG_{11} \cdot L_S}{MG_{11}}\right\}MG_{11}, \quad rG_1=w,$$

which is evaluated for the Cobb-Douglas technology in the following way.

$$S^* = \left\{\frac{nwM \cdot mrK}{(\alpha^* - 1)M} + \frac{mrK \cdot mwL_S(\alpha^* - 1)}{M(\alpha^* - 1)}\right\}\frac{MG_{11}}{mwK} > 0.$$

Thus, S^* commonly proves to be positive in the present day economy, where $nwM > mwL_S(1-\alpha^*)$. So does S since $-m^2H_{11}F_1F_2(p^*U_{12}-wU_{22})>0$. After all these offsetting operations, three positive terms, i.e., the first terms of the first and the fifth groups, and the second term of the final group remain. Hence, it has been shown that dL/dI proves to be negative since A(<)0.

References

Becker, G. (1965) A theory of the allocation of time, *Economic Journal*, 75, 493-517.

Bryant, W.K., Zick, C.D., and Kim, H. (1992) *The Dollar Value of Household Work*, Cornell University.

Bureau of Economic Analysis, U.S. Department of Commerce (1976-98) *National Income and Product Accounts*.

Bureau of Labor Statistics, U.S. Department of Labor (1976-98) *Current Population Survey*.

Economic Planning Agency, Government of Japan (1976-98) *Kokumin Keizai Keisan Nenpo* (in Japanese) (*Annual Reports on National Accounts*).

——— (1997) *What Is The Price of Your Household Work? A Report on the Money Valuation of Unpaid Labor* (in Japanese).

Eisner, R. (1989) *The Total Incomes System of Accounts*, Univ. Chicago Press, Chicago.

Greenwood, J., and Hercowitz, Z. (1991) The allocation of capital and time over the business cycle, *Journal of Political Economy*, 99, 1188-1214.

Maruyama, Y. (1984) *Kigyo-Kakei Fukugotai no Riron* (in Japanese) (*A Theory of Firm-Household Complex*), Sobunsha, Tokyo.

——— (1994) The business-cyclic overwork and the sociostructural overwork (in Japanese), The Japan Society of Home Economics, Summary Proceedings of the 46th Annual Meeting, p. 81.

——— (1996) Behavior of the economy that necessitates household work (in Japanese), *J. Family and Consumer Economics of Japan*, 9, 2-8.

Maruyama, Y. and Sasaki, K., eds. (1998) *Studies in Family Economy and Aggregate Fluctuations*, Univ. Tsukuba Library.

Singh, I., Squire, L. and Strauss, J. eds. (1986) *Agricultural Household Models*, Johns Hopkins Univ. Press, Baltimore.

Sonoda, T. and Maruyama, Y. (1999) Effects of the internal wage on output supply: a structural estimation for Japanese rice farmers, *Amer. J. of Agr. Econ.*, 81, 131-143.

——— and ——— (2000) The behavior of agricultural households under constrained off-farm wage employment: an alternative decomposition of their comparative statics, *J. Rural Econ*, 72, 109-121.

Statistics Bureau, Management and Coordination Agency, Government of Japan (1976-98) *Rodoryoku Chosa Nenpo* (in Japanese) (*Annual Reports on the Labor Force Survey*).

Yoka Kaihatsu Center (Leisure Development Center)(1978) *A Study in the Demand for Leisure in Japan: Household and the Economy* (in Japanese), Tokyo.